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Multi-Response Optimization of Friction Stir Welding of AA2050 Using Response Surface Methodology Coupled with Grey Relational Analysis and Principal Component Analysis

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The third generation aluminium-lithium alloy AA2050 finds wide applications in defence and aircraft industries by virtue of its high strength-to-weight ratio and excellent corrosion resistance. Friction stir welding (FSW), relatively novel technique, is more suitable to join this alloy compared to other conventional fusion welding techniques. In this work, the overall quality of the weld joint was decided from the higher values of tensile strength, yield strength, percentage elongation, hardness of weld zone, hardness of heat affected zone, bending load and lower value of width of heat affected zone. The optimal (combined) design was used to design the experiments with four numeric factors (traverse speed, rotational speed, tilt angle and shoulder diameter) and a categoric factor (tool pin profile). The multi-response optimization problem was reduced into a single-response optimization problem using grey relational analysis (GRA); principal component analysis (PCA) was used to assign optimal weighting values for the responses in the process of dimensionality reduction. Mathematical model for the reduced single response, which can be perceived as overall weld quality, was developed by the response surface methodology (RSM) and the optimization of process parameters was also carried by the RSM. Analysis of variance (ANOVA) was carried to evaluate the significance of each parameter on the overall weld quality and the adequacy of the developed model. The confirmation tests conducted at optimum levels of parameters proved the effectiveness and robustness of the method.

Keywords: Multi-response optimization, grey relational analysis, principal component analysis, response surface methodology, analysis of variance

1 Introduction

The Welding Institute (TWI) Ltd invented and developed a solid state joining technique known as Friction stir welding (FSW). It has been evolved as more suitable welding technique for the aluminium alloys. The weld joints of aluminium alloys prepared by the conventional fusion welding techniques encounter the defects such as hot cracking, distortion, solidification shrinkage, embrittlement due to dissolved gases and high residual stress. FSW gives dimensional stability and better mechanical properties resulted from dynamic recrystallization of microstructure. FSW is also a green technique as the process is energy efficient, produces no fumes and does not use shielding gas [1, 2]. Advancement of FSW with the well-developed tools facilitated wider usage of this joining method in aerospace, ship building and automotive industries. FSW is very popular technique to join the aluminium alloys of 2000, 5000, 6000, 7000, Al-Li series and aluminium matrix composites [3].

This work is aimed to develop a regression model for the overall weld quality and optimize the weld parameters of FSW of third generation Al-Li alloy AA2050 – T84 (Solution heat treated, cold worked and artificially underaged). The base metal have Ultimate tensile strength of 585 MPa, Yield strength of 565 MPa and percentage elongation of 8% [4].

Tab. 1 *Chemical composition of AA2050 (weight %)*

Si	Fe	Cu	Mg	Mn	Li	Ti	Zn	Zr	Ag	Al
0.0354	0.0511	3.53	0.358	0.345	0.85	0.0375	0.0328	0.0868	0.363	94.2

In the traditional response surface methodology (RSM) approach, responses were optimized individually [5-15]. A.Heidarzadeh et al., [5] manufactured the friction stir welded joints of aluminium alloy AA6061-T4 with different combinations of welding speed, rotational speed and axial force. The optimum values of above process parameters for maximum tensile strength and the optimum proces parameters for maximum percentage elongation were evaluated separately. Morteza Ghaffarpour et al., [6] studied the optimization of FSW of dissimilar aluminium alloys AA5083 and AA6061 sheets. Optimum values of traverse speed, rotational speed, shoulder diameter and tool pin diameter for maximum tensile strength were calculated. R.Palanivel et al., [7] evaluated the effect of traverse speed, rotational speed, tool pin profile, and axial force on ultimate tensile strength of weld joint of dissimilar aluminium alloys AA5083-H111 and AA6351-T6. R.Kadaganchi et al., [8] carried FSW of aluminium alloy AA2014-T6 with the process parameters rotational speed, traverse speed, tilt angle and too pin profile. The optimum parametric combinations for maximum tensile strength, maximum yield strength and maximum percentage elongation were evaluated separately. A.Farzadi et al., [9] found optimum values of traverse speed, rotational speed, pin diameter and shoulder diameter of FSW of aluminium alloy AA7075-T6 to maximize the tensile strength of the joint. B.Ravi Sankar et al., [10] aimed to optimize the process parameters (traverse speed, rotational speed, pin diameter) of weld joint of aluminium alloy AA6061 for higher tensile strength and hardness separately. Yordi Kristianto Budiono et al., [11] developed a model for shear strength of Deep Drawing Steel (DDS), which is joined by resistrance spot welding. They considered thickness of plate, welding current and welding time as process parameters. A.Goyal et al., [12] investigated the influence of process parameters rotational speed, traverse speed, tool shoulder diameter, tool hardness, tool pin profile and tool tilt angle on inter-granular corrosion rate of FSW joint of aluminium alloy AA5086-H32. Optimum set of the parameters were estimated to minimize the corrosion rate. Ali Habibizadeh et al., [13] designed and carried the friction stir spot welding of AA1050 and C10100 with tool rotational speed, dwell time and tool plunge depth as control parameters. They used the RSM to design the experiments and ANOVA to identify the main affecting parameters in maximizing the shear failure load. Hangyan Guo et al., [14] used the RSM to design the experiments and develop a second order model for surface roughness in terms of tool tip radius, feed rate, discrete angle and cutting depth. Nor Ana Rosli et al., [15] determined the optimum input parameters (welding speed, wire feeding speed and pulse) of plasma welding for high deposition rate in an additive manufacturing structure.

To overcome the limitation of traditional RSM approach, Shanavas et al., [16] combined the grey relational analysis (GRA) and RSM to solve the multi-response optimization problem of FSW of AA5052-H32. The optimal values of traverse speed, rotational speed, tilt angle and tool pin profile to maximize both the tensile strength and the hardness were evaluated. The incorporation of GRA facilitates the conversion of multi-response system into single-response system. The resulted single response was called as grey relational grade (GRG). However the weights of responses to calculate the GRG were taken as equal. When the weights are decided by engineering judgement or decided by the past experience, leads to introduction of subjectivity into the problem. The introduction of principal component analysis (PCA) to calculate the optimum weights of responses ensures the objectivity [17-20]. The GRA-PCA-RSM hybrid technique has been implemented successfully in many areas of research work. Manish Gangil et al., [17] solved the multi-response optimization problem of electrical discharge machining of Ti-6Al-4V using this hybrid technique. Suneel Kumar et al., [18] used the same hybrid technique to find the optimum parameter values of CNC turning of aluminium alloy AA6463 to minimize the arithmetical mean roughness (R_a) value and mean roughness depth (R_z). Raju et al., [19] implemented the same hybrid method to maximize the tensile strength, yield strength and toughness of TIG weld joints of Incoloy (800HT). Welding current, voltage and welding speed were selected parameters for the study. Vijayan et al., [20] applied the hybrid method to solve the multi-response optimization problem of FSW of dissimilar aluminium alloys AA2024 and AA6061. Rotational speed, traverse speed, axial load and tool pin profile were the parameters considered and analysis was carried to maximize the tensile properties.

The optimization of the process parameters of FSW of third generation Al-Li alloy AA2050 was hardly taken up. The hybrid technique GRA-PCA-RSM was successfully applied in various areas of research but scarcely used in the FSW area, hence the hybrid approach was selected for the multi-response optimization in this work. After having scrupulous examination of earlier FSW works on aluminium alloys, the prominent process parameters namely traverse speed, rotational speed, tilt angle, shoulder diameter and tool pin profile were selected for experimentation. To make the investigation comprehensive, more number of responses was considered in this study. The responses considered were tensile strength, yield strength, percentage elongation, hardness of weld zone, hardness of heat affected zone, bending load and width of heat affected zone.

2 Materials and methods

2.1 Equipment for experimentation

Third generation aluminium-lithium alloy AA2050-T84 plates of dimension 200X100X4mm were butt welded by FSW in a single pass. The plates were clamped in required position using special fixtures. The tools were made of AISI H13 tool steel. Sixteen tools with four different shoulder diameters and four different tool pin profiles were used. The probe length of pin for all the tools was 3.8 mm. The FSW was carried on FN2EV knee type milling machine made by HMT Ltd., Pinjore, India.

2.2 Process parameters range and their levels

The limits of process parameters were selected by conducting several trails and from the data available in the literature for the FSW of other aluminium alloys. The initial design run was carried at 100mm/min of traverse speed, 1400rpm of rotational speed, 1^0 of tool tilt angle, 20mm of shoulder diameter and with taper square tool pin. The process parameters selected and their levels are furnished in the table 2. The geometry of different tool pin profiles is shown in the figure 1.

***Tab. 2** Process parameters range and their levels*

S.NO	PROCESS PARAMETER	ACRONYM	UNITS	RANGE			
				LEVEL1	LEVEL 2	LEVEL 3	LEVEL 4
1	Traverse Speed	TS	mm/min	80	100	125	160
2	Rotational Speed	RS	rpm	900	1120	1400	1800
3	Tilt Angle	TA	degrees	0.5	1	1.5	2
4	Shoulder Diameter	SD	mm	16	18	20	22
5	Tool Pin Profile	TPP	_	SCL	TCL	SSQ	TSQ

SCL-Straight Cylindrical, TCL-Taper Cylindrical, SSQ-Straight Square, TSQ-Taper Square

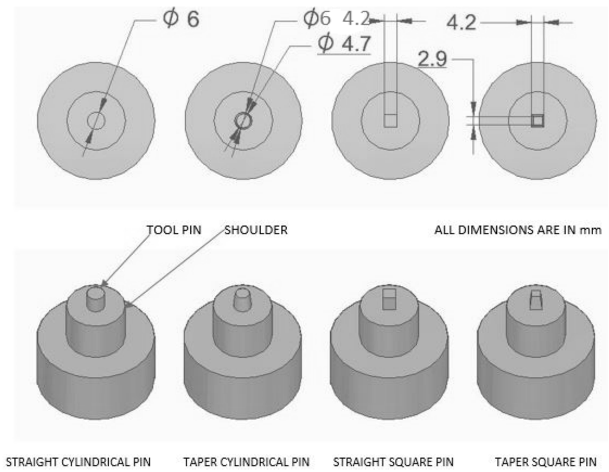

***Fig. 1** Geometry of tool pin profiles*

2.3 Design of experiments

The experiments were designed by RSM based optimal (combined) design, which is a flexible design that accommodates both categoric factors and numeric factors. In this work, four numeric factors (TS, RS, TA and SD) and one categoric factor (TPP) were involved. For four numeric factors sixteen (2^4) factorial points, five lack-of-fit points and five replicates at the central points were used for the design. Hence, the total number of experimental runs required was 26, which are shown in the table 3.

2.4 Evaluation of the responses

Tensile tests on the joints were carried on universal testing machine FIE-UTES40 by following the standards IS1608(Part-1):2018. Hardness tests were carried on Vickers hardness testing machine with a load of 5kgf (HV5) by following the standards IS1501(Part-1):2020. Bending test was carried with a mandrel of 25 mm diameter on universal testing machine with a bending jig by following the standards IS1599:2019. Inverted metallurgical microscope (DEWINTER-1500X) was used to measure the width of the heat affected zone. The test results for all the runs are furnished in the table 3. The macrographs of the weld beads obtained from the experimentation are presented in the figure 2 and figure 3.

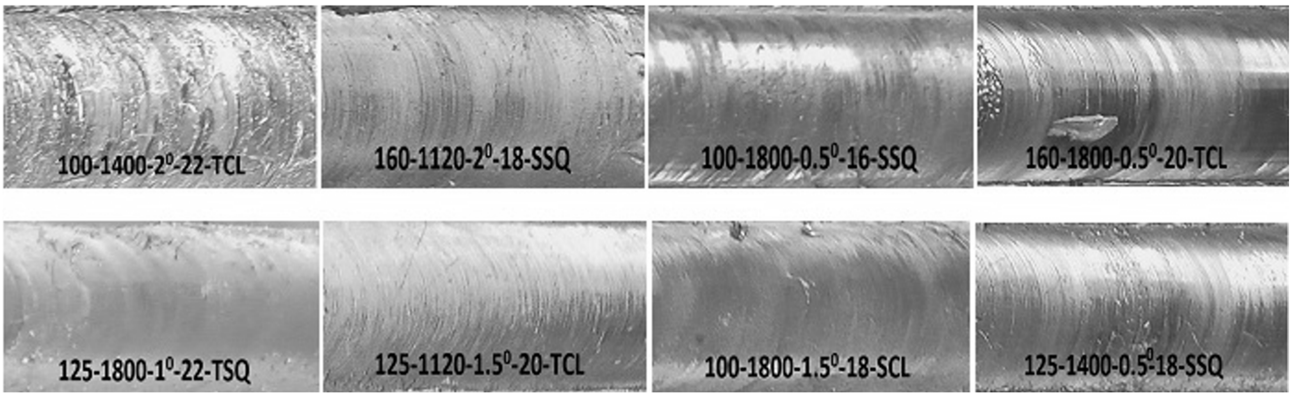

***Fig. 2** Macrographs of weld beads obtained from the experimentation [TS–RS–TA–SD–TPP]*

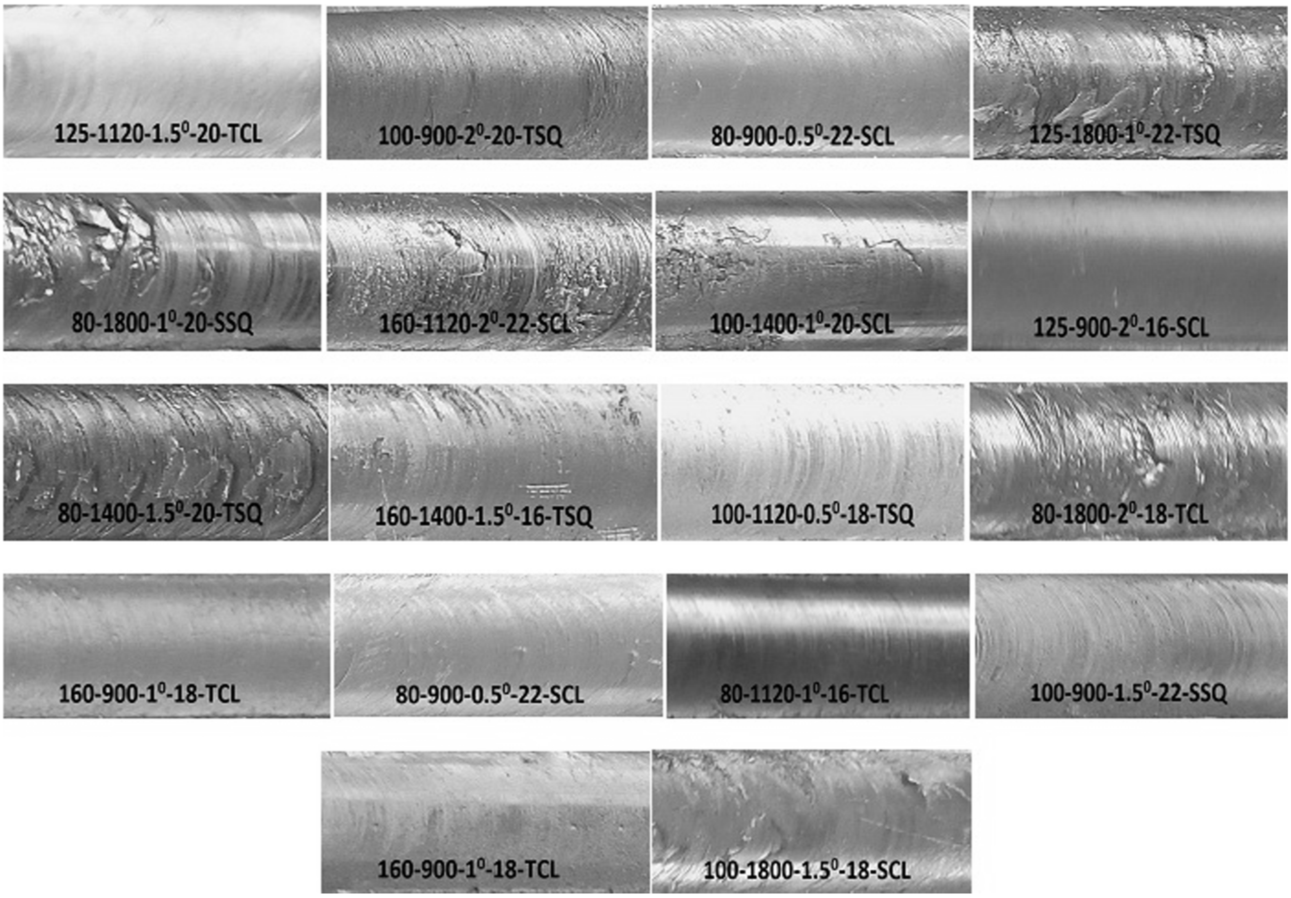

Fig. 3 *Macrographs of weld beads obtained from the experimentation [TS–RS–TA–SD–TPP]*

Tab. 3 *Experimental runs and the results*

RUN	TS	RS	TA	SD	TPP	Tensile Strength (MPa)	Yield Strength (MPa)	% Elonga-tion	Hardness of Weld Zone	Hardness of HAZ	Bending Load (N)	Width of HAZ μm
1	100	1400	2	22	TCL	259.39	228.28	2.20	105.5	114	3260	1974.133
2	160	1120	2	18	SSQ	336.35	240.74	4.30	102	119	1200	1645.269
3	100	1800	0.5	16	SSQ	224.80	209.14	2.56	105.5	111	1360	1074.459
4	160	1800	0.5	20	TCL	181.94	160.98	1.84	105.75	105	1460	1348.519
5	125	1800	1	22	TSQ	221.25	201.37	2.06	105.75	111.25	1860	1591.777
6	125	1120	1.5	20	TCL	296.10	240.27	2.28	106.5	111.9	2180	1504.893
7	100	1800	1.5	18	SCL	244.20	211.46	2.26	107.5	109.75	1280	1624.789
8	125	1400	0.5	18	SSQ	273.53	255.49	3.36	96.5	120	1240	1004.7
9	125	1120	1.5	20	TCL	281.61	236.71	2.34	107	110.75	1860	1582.537
10	100	900	2	20	TSQ	335.63	255.51	2.26	101	112.25	2980	1744.238
11	80	900	0.5	22	SCL	291.70	238.31	3.20	103	112.25	1520	1994.353
12	125	1800	1	22	TSQ	230.03	210.16	1.98	107	111	1520	1656.54
13	80	1800	1	20	SSQ	228.74	200.70	2.86	105	116	960	1648.51
14	160	1120	2	22	SCL	326.73	232.61	3.96	108.5	111	1460	1763.343
15	100	1400	1	20	SCL	280.08	226.78	2.96	103	103	1080	1598.176
16	125	900	2	16	SCL	355.71	252.03	2.10	103.75	107.75	1610	1338.095
17	80	1400	1.5	20	TSQ	226.67	213.33	1.86	105	108.5	2620	1984.864
18	160	1400	1.5	16	TSQ	240.54	227.41	1.52	107.5	117.25	2460	1252.413
19	100	1120	0.5	18	TSQ	291.62	271.34	2.06	108	115.25	3180	1279.056
20	80	1800	2	18	TCL	226.27	213.19	1.88	107	110.75	1260	1566.674
21	160	900	1	18	TCL	278.42	207.53	2.56	101.5	117.75	1780	1394.903
22	80	900	0.5	22	SCL	284.45	224.65	3.12	102.5	111.75	1380	2054.387
23	80	1120	1	16	TCL	250.75	238.91	2.72	106	112.25	1840	1262.41
24	100	900	1.5	22	SSQ	340.85	255.02	4.20	98.5	120.5	1280	1863.645
25	160	900	1	18	TCL	260.39	216.54	2.62	102.75	115.75	1940	1414.534
26	100	1800	1.5	18	SCL	239.35	219.95	2.32	107	105.25	1040	1604.986

2.5 Calculation of grey relational coefficients

Grey relational analysis (GRA) is a analyzing technique applied to incomplete data with multiple variables to make a decision. System with relatively fewer data with insufficient description, for which standard statistical assumptions are not applicable is known as grey system [21]. Multi-response problems with contradicting performance targets contains lot of uncertainty, hence such problems have to be solved by the grey system theory.

In the GRA as a first step, to overcome the inconvenience caused by different dimensions of response variables and to simplify the calculation, normalization of the response data has to be done. The following formulae shown in the equations (1), (2) and (3) are used to normalize the data based on the type of performance target for response.

Larger-the-better or benefit type:

$$xs_i(j) = \frac{|x_i(j) - minx_i(j)|}{maxx_i(j) - minx_i(j)} \quad (1)$$

Smaller-the-better or cost type:

$$xs_i(j) = \frac{maxx_i(j) - x_i(j)}{maxx_i(j) - minx_i(j)} \quad (2)$$

Nominal the best type:

$$xs_i(j) = 1 - \frac{|x_i(j) - x(j)|}{max(maxx_i(j) - x(j), x(j) - minx_i(j))} \quad (3)$$

Where:

$x_i(j)$…The measured value of the particular run,

$maxx_i(j)$…The maximum value of the reponse of all the runs,

$minx_i(j)$…The minimum value of the reponse of all the runs,

$x(j)$…The target value in the nominal the best type of response.

Later the difference sequence is obtained from the normalised data using the equation (4):

$$\Delta x_i(j) = |x_0(j) - xs_i(j)| \quad (4)$$

Where:

$\Delta x_i(j)$…The difference sequence, which is the absolute difference of the reference sequence $x_0(j)$ is and the normalised sequence $xs_i(j)$.

Grey relational coefficients are determined using the equation (5):

$$\xi_i(j) = \frac{\Delta min + p\Delta max}{\Delta x_i(j) + p\Delta max} \quad (5)$$

Where:

Δmax and Δmin…The maximum and minimum values of difference sequence,

p…The distinguishing coefficient, whose value is usually taken as 0.5.

2.6 Identification of principal components

Principal Component Analysis (PCA) is carried to reduce the dimensionality of large number of interrelated variables to small number of uncorrelated variables known as principal components. The reatined first few principal components explains most of the variation present in the data [22].

In this work, the grey correlation coefficient matrix of responses is considered as decision matrix.

$$X_i = \begin{bmatrix} X_{11} & X_{12} & \dots & X_{1n} \\ X_{21} & X_{22} & \dots & X_{2n} \\ X_{31} & X_{32} & \dots & X_{3n} \\ \vdots & \vdots & \vdots & \vdots \\ X_{m1} & X_{m2} & \dots & X_{mn} \end{bmatrix} \quad (6)$$

Where:

X_i…The response,

n…The number of the responses measured,

m…The number of experiments.

Correlation coefficient matrix is defined as shown in the equation (7):

$$A_{kl} = \left[\frac{Cov(X_i(k), X_i(l))}{\sigma_{Xi}(k)\sigma_{Xk}(l)}\right] \quad (7)$$

Where:

$Cov(X_i(k), X_i(l))$ …The covariance sequence of $X_i(k)$ and $X_i(l)$,

$\sigma_{Xi}(k)$ and $\sigma_{Xk}(l)$…The standard deviations of sequences of $X_i(k)$ and $X_i(l)$ respectively.

The relation between correlation matrix, Eigen values and Eigen vector is shown in the equation (8), from which we can compute the Eigen values and Eigen vectors.

$$[A - \lambda I]V = 0 \quad (8)$$

Where:

I…Identity matrix,

V…The Eigen vector,

λ…The Eigen value,

A…The correlation coefficient matrix.

Later principal components are calculated using the equation (9):

$$PC_i = \sum_{i=1}^{n} X_m(i)V \quad (9)$$

Where:

PC_i…The principal component.

The number of principal compents will be equal to the number of responses. The principal components with Eigen value more than one, usually explains the most of the variance in the data, hence are retained. After identifying the principal components, weighted principal component values (t- values) are obtained from equation (10):

$$t = \sum_{k=1}^{m} w_k PC_k \quad (10)$$

Weights of the principal components (w_k) are computed from the Eigen values ($\lambda_1, \lambda_2, \ldots\ldots, \lambda_n$) using the following relations.

$$w_{k1} = \frac{\lambda_1}{\lambda_1 + \lambda_2 + \cdots + \lambda_n} \quad (11)$$

$$w_{k2} = \frac{\lambda_2}{\lambda_1 + \lambda_2 + \cdots + \lambda_n} \quad (12)$$

$$w_{kn} = \frac{\lambda_n}{\lambda_1 + \lambda_2 + \cdots + \lambda_n} \quad (13)$$

The sign of the majority of the components decides the sign of w_k . The matrix multiplication of t-values and grey relational coefficients yields grey relational grades, which are perceived as the overall weld quality of the joints.

2.7 Development of mathematical model and optimization

Response surface methodology (RSM) is a collection of regression and statistical techniques used to develop an empirical model, analyse and optimize the multi variable problems. It also used to find the interaction effect of the significant factors [23].

After deciding the range and levels of process parameters, the experiments are designed. In this work, four numerical factors and one categorical factor were chosen and 26 experimental runs were designed. From the experimental data a regression model for overall weld quality has to be developed in terms of process parameters. For each tool pin profile, the second order polynomial equation for overall weld quality is developed as shown in the equation (14).

$$X = \beta_o + \sum_{i=1}^{n} \beta_i Y_i + \sum_{i=1}^{n} \beta_{ii} Y_i^2 + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \beta_{ij} Y_i Y_j + \varphi \quad (14)$$

Where:

X…The predicted response,

n…The number of independent variables,

Y_i and Y_j…Coded values of the process parameters,

β_o...The coefficient of constant,

β_i, β_{ii}, and β_{ij}…Coefficient of linear term, coefficient of quadratic term and coefficient of interaction term respectively,

φ…Random error.

The adequacy of the developed model and significance of process parameters are found with the analysis of variance (ANOVA). ANOVA and identification of optimum levels of process parameters are performed by the Design Expert 12 software.

3 Results and discussion

3.1 Grey Relational Analysis

Tab. 4 *Normalized sequence*

NORMALIZED SEQUENCE							
RUN	Tensile Strength	Yield Strength	% Elongation	Hardness of Weld Zone	Hardness of HAZ	Bending Load	Width of HAZ
1	0.4457	0.6098	0.2446	0.7500	0.6286	1.0000	0.0765
2	0.8886	0.7227	1.0000	0.4583	0.9143	0.1043	0.3898
3	0.2466	0.4364	0.3741	0.7500	0.4571	0.1739	0.9335
4	0.0000	0.0000	0.1151	0.7708	0.1143	0.2174	0.6725
5	0.2262	0.3660	0.1942	0.7708	0.4714	0.3913	0.4407
6	0.6570	0.7185	0.2734	0.8333	0.5086	0.5304	0.5235
7	0.3583	0.4574	0.2662	0.9167	0.3857	0.1391	0.4093
8	0.5271	0.8564	0.6619	0.0000	0.9714	0.1217	1.0000
9	0.5736	0.6862	0.2950	0.8750	0.4429	0.3913	0.4495
10	0.8844	0.8566	0.2662	0.3750	0.5286	0.8783	0.2955
11	0.6316	0.7007	0.6043	0.5417	0.5286	0.2435	0.0572
12	0.2767	0.4456	0.1655	0.8750	0.4571	0.2435	0.3790
13	0.2693	0.3599	0.4820	0.7083	0.7429	0.0000	0.3867
14	0.8332	0.6491	0.8777	1.0000	0.4571	0.2174	0.2773
15	0.5648	0.5962	0.5180	0.5417	0.0000	0.0522	0.4346
16	1.0000	0.8250	0.2086	0.6042	0.2714	0.2826	0.6824
17	0.2574	0.4744	0.1223	0.7083	0.3143	0.7217	0.0662
18	0.3372	0.6019	0.0000	0.9167	0.8143	0.6522	0.7640
19	0.6312	1.0000	0.1942	0.9583	0.7000	0.9652	0.7386
20	0.2551	0.4731	0.1295	0.8750	0.4429	0.1304	0.4646
21	0.5552	0.4218	0.3741	0.4167	0.8429	0.3565	0.6283
22	0.5899	0.5769	0.5755	0.5000	0.5000	0.1826	0.0000
23	0.3960	0.7061	0.4317	0.7917	0.5286	0.3826	0.7545
24	0.9145	0.8521	0.9640	0.1667	1.0000	0.1391	0.1817
25	0.4515	0.5034	0.3957	0.5208	0.7286	0.4261	0.6096
26	0.3304	0.5343	0.2878	0.8750	0.1286	0.0348	0.4281

***Tab. 5** Deviational sequence*

DEVIATIONAL SEQUENCE							
RUN	Tensile Strength	Yield Strength	% Elongation	Hardness of Weld Zone	Hardness of HAZ	Bending Load	Width of HAZ
1	0.5543	0.3902	0.7554	0.2500	0.3714	0.0000	0.9235
2	0.1114	0.2773	0.0000	0.5417	0.0857	0.8957	0.6102
3	0.7534	0.5636	0.6259	0.2500	0.5429	0.8261	0.0665
4	1.0000	1.0000	0.8849	0.2292	0.8857	0.7826	0.3275
5	0.7738	0.6340	0.8058	0.2292	0.5286	0.6087	0.5593
6	0.3430	0.2815	0.7266	0.1667	0.4914	0.4696	0.4765
7	0.6417	0.5426	0.7338	0.0833	0.6143	0.8609	0.5907
8	0.4729	0.1436	0.3381	1.0000	0.0286	0.8783	0.0000
9	0.4264	0.3138	0.7050	0.1250	0.5571	0.6087	0.5505
10	0.1156	0.1434	0.7338	0.6250	0.4714	0.1217	0.7045
11	0.3684	0.2993	0.3957	0.4583	0.4714	0.7565	0.9428
12	0.7233	0.5544	0.8345	0.1250	0.5429	0.7565	0.6210
13	0.7307	0.6401	0.5180	0.2917	0.2571	1.0000	0.6133
14	0.1668	0.3509	0.1223	0.0000	0.5429	0.7826	0.7227
15	0.4352	0.4038	0.4820	0.4583	1.0000	0.9478	0.5654
16	0.0000	0.1750	0.7914	0.3958	0.7286	0.7174	0.3176
17	0.7426	0.5256	0.8777	0.2917	0.6857	0.2783	0.9338
18	0.6628	0.3981	1.0000	0.0833	0.1857	0.3478	0.2360
19	0.3688	0.0000	0.8058	0.0417	0.3000	0.0348	0.2614
20	0.7449	0.5269	0.8705	0.1250	0.5571	0.8696	0.5354
21	0.4448	0.5782	0.6259	0.5833	0.1571	0.6435	0.3717
22	0.4101	0.4231	0.4245	0.5000	0.5000	0.8174	1.0000
23	0.6040	0.2939	0.5683	0.2083	0.4714	0.6174	0.2455
24	0.0855	0.1479	0.0360	0.8333	0.0000	0.8609	0.8183
25	0.5485	0.4966	0.6043	0.4792	0.2714	0.5739	0.3904
26	0.6696	0.4657	0.7122	0.1250	0.8714	0.9652	0.5719

***Tab. 6** Grey relational coefficients and grey relational grades*

RUN	Tensile Strength	Yield Strength	% Elongation	Hardness of Weld Zone	Hardness of HAZ	Bending Load	Width of HAZ	Overall Weld Quality (GRG)
1	0.4743	0.5617	0.3983	0.6667	0.5738	1	0.3512	0.863079
2	0.8178	0.6433	1	0.48	0.8537	0.3583	0.4503	1.220316
3	0.3989	0.4701	0.4441	0.6667	0.4795	0.377	0.8827	0.522075
4	0.3333	0.3333	0.361	0.6857	0.3608	0.3898	0.6042	0.413398
5	0.3925	0.4409	0.3829	0.6857	0.4861	0.451	0.472	0.575626
6	0.5931	0.6398	0.4076	0.75	0.5043	0.5157	0.512	0.787391
7	0.4379	0.4796	0.4052	0.8571	0.4487	0.3674	0.4584	0.568071
8	0.5139	0.7769	0.5966	0.3333	0.9459	0.3628	1	0.937901
9	0.5397	0.6144	0.4149	0.8	0.473	0.451	0.476	0.72362
10	0.8123	0.7771	0.4052	0.4444	0.5147	0.8042	0.4151	1.112796
11	0.5758	0.6256	0.5582	0.5217	0.5147	0.3979	0.3465	0.868157
12	0.4087	0.4742	0.3747	0.8	0.4795	0.3979	0.446	0.567457
13	0.4063	0.4386	0.4912	0.6316	0.6604	0.3333	0.4491	0.64899
14	0.7499	0.5876	0.8035	1	0.4795	0.3898	0.4089	0.916951
15	0.5346	0.5532	0.5092	0.5217	0.3333	0.3453	0.4693	0.712609
16	1	0.7408	0.3872	0.5581	0.407	0.4107	0.6115	1.008363
17	0.4024	0.4875	0.3629	0.6316	0.4217	0.6425	0.3487	0.658571
18	0.43	0.5568	0.3333	0.8571	0.7292	0.5897	0.6794	0.662033
19	0.5755	1	0.3829	0.9231	0.625	0.935	0.6567	1.019526
20	0.4016	0.4869	0.3648	0.8	0.473	0.3651	0.4829	0.550147
21	0.5292	0.4637	0.4441	0.4615	0.7609	0.4373	0.5736	0.767927
22	0.5494	0.5417	0.5409	0.5	0.5	0.3795	0.3333	0.805701
23	0.4529	0.6298	0.468	0.7059	0.5147	0.4475	0.6707	0.694386
24	0.8539	0.7717	0.9329	0.375	1	0.3674	0.3793	1.350589
25	0.4769	0.5017	0.4528	0.5106	0.6481	0.4656	0.5615	0.73314
26	0.4275	0.5178	0.4125	0.8	0.3646	0.3412	0.4665	0.564301

Normalization of the response values was carried according to the equations (1) and (2) and the values are shown in the table 4. The experimental results of tensile strength, yield strength, percentage elongation, hardness and bending load were normalized considering their performance target as 'larger-the-better' or 'benefit' type. The values of width of the heat affected zone was normalized considering its performance target as 'smaller-the-better' or 'cost type'. The normalization process reduced the respective response values to be between 0 and 1. The deviational sequences were computed using the equation (4) and the values are shown in the table 5. After identifying the minimum and maximum values of deviational sequences for each response, the grey relational coefficients were calculated using the equation (5) and the coefficients are shown in the table 6. The distinguishing factor for calculation of grey relational coefficients was taken as 0.5.

3.2 Principal Component Analysis

Principal component analysis (PCA) was carried to find the weighting factors for responses in order to remove subjectivity in assigning the weightings to the responses. The grey relational component matrix calculated in the previous section and furnished in table 6 was utilised as requisite data for the PCA. Using the equation (7) the correlation matrix was evaluated and was substituted in the equation (8) to compute Eigen values. The Eigen values and their explained variations are furnished in the table 7. Based on these Eigen values the significant principal components were identified. The PCA was carried using the software IBM SPSS Statistics 26.

Tab 7. *Eigen values and explained variation*

Component	Eigen value	% of Variance	Cumulative % of variance
1	2.643	37.754	37.754
2	1.502	21.456	59.210
3	1.240	17.708	76.917
4	0.723	10.333	87.251
5	0.591	8.449	95.699
6	0.156	2.222	97.922
7	0.145	2.078	100.000

In this work seven variables were analysed, hence seven principal components were extracted. The Eigen values of first three principal components were more than unity; hence they were identified as significant and retained for further calculations. The first three principal components accounted for 76.917% of the total variance. Weighted principal component values which are known as t-values were calculated using the equation (10). The weights of the three retained principal components were calculated from the Eigen values of the components using the equation (11) and the weights were obtained as 0.491, 0.279 and -0.230 respectively. Grey relational grades (GRG) were obtained from matrix multiplication of grey relational coefficients matrix and the t-value matrix and are shown in the table 6. The GRG represents the overall quality of the weld joint, hence now onwards called as overall weld quality.

Tab. 8 *Component matrix of principal components*

	Component Matrix			
	PC1	PC2	PC3	t-values
Tensile Strength	0.784	0.202	-0.365	0.5246234
Yield Strength	0.675	0.636	0.100	0.4860408
% Elongation	0.783	-0.368	-0.262	0.3407421
Hardness of weld	-0.614	0.325	-0.139	-0.1779254
Hardness of HAZ	0.760	-0.107	0.369	0.2577381
Bending Load	-0.021	0.896	-0.002	0.2411141
Width of HAZ	0.062	0.000	0.934	-0.1845377

3.3 Response Surface Methodology

By performing backward elimination of insignificant terms, reduced quadratic models for the overall weld quality were developed. The regression models for the tool pins of straight cylinder, taper cylinder, straight square and taper square are shown in equations (16), (17), (18) and (19) respectively. The regression models are in terms of process parameters with their original units.

Tool Pin profile: **SCL**

Overall Weld Quality = 0.570078+0.00281134*TS-0.000610941*RS+0.545719*TA+0.0144024*SD-0.00529394*Traverse Speed*TA+0.0198099*TA*SD (16)

Tool Pin Profile: **TCL**

Overall Weld Quality = 0.671145+0.0036325*TS-0.000571573*RS+0.249172*TA+0.0144024* SD-0.00529394*TS*TA+0.0198099*TA*SD (17)

Tool Pin Profile: **SSQ**

Overall Weld Quality = -0.165056+0.0108389*TS-0.000403699*RS+0.412492*TA+0.0144024* SD-0.00529394*TS*TA+0.0198099*TA*SD (18)

Tool Pin Profile: **TSQ**

$$\text{Overall Weld Quality} = 0.856551+0.010197*TS-0.00093173*RS+0.0276307*TA+0.0144024*SD-0.00529394*TS*TA+0.0198099*TA*SD \quad (19)$$

3.4 Analysis of variance

Analysis of variance (ANOVA) is used to test the adequacy of the model developed for the overall weld quality and also to find the significance of each process parameter. The adequacy and significance of the developed model is decided by the p-value, F-value and the fit statistics. The statistical software Design Expert 12 was used to carry the ANOVA. The large F-value of the model (96.17) and the very small value of p-value (less than 0.0001) implied that the model was adequate and significant. There was less than 0.01% chance that F-value of this magnitude occurs due to noise. The following parameters and their interactions B (RS), C (TA), D (SD), E (TPP), AC, AE, BE and CE were significant as their p-values were less than 0.05. The lack of fit of the model is not significant as its F-value is very small (0.3148) and p-value is very large (0.7434). There are 74.34% of chances are there that lack of fit values of this magnitude can occur due to noise. The results of ANOVA are shown in the table 9. The fit statistics obtained from the ANOVA for the models are furnished in the table 10.

Tab. 9 *Analysis of variance*

Source	Sum of Squares	df	Mean Square	F-value	p-value		% Contribution
Model	1.29	18	0.0716	96.17	< 0.0001	significant	
A-Traverse Speed	0.0001	1	0.0001	0.0817	0.7833		0.01
B-Rotational Speed	0.5919	1	0.5919	794.9	< 0.0001	significant	67.32
C-TA	0.0052	1	0.0052	7	0.0332	significant	0.59
D-Shoulder Diameter	0.0418	1	0.0418	56.18	0.0001	significant	4.76
E-TPP	0.3179	3	0.106	142.3	< 0.0001	significant	12.05
AC	0.0135	1	0.0135	18.18	0.0037	significant	1.54
AE	0.0336	3	0.0112	15.06	0.0019	significant	1.28
BE	0.0383	3	0.0128	17.15	0.0013	significant	1.45
CD	0.0021	1	0.0021	2.88	0.1333		0.24
CE	0.0684	3	0.0228	30.62	0.0002	significant	2.59
Residual	0.0052	7	0.0007				
Lack of Fit	0.0006	2	0.0003	0.3148	0.7434	not significant	
Pure Error	0.0046	5	0.0009				
Cor Total	1.29	25					

Tab. 10 *Fit statistics*

Standard deviation	0.0273	R^2	0.9960
Mean	0.7790	Adjusted R^2	0.9856
C.V. %	3.5	Predicted R^2	0.8984
		Adequate Precision	40.2973

The coefficient of determination (R^2) of the model was 0.996, which indicates that the model was significant. The predicted R^2 (0.8984) was in reasonable agreement with the adjusted R^2 (0.9856) as the difference between them was less than 0.2. Adequate precision ratio was a measure of signal-to-noise ratio and a value of more than 4 is preferable. The model can be used to steer through the design space as the adequate precision ratio of 40.297 ensures adequate signal. Small value of standard deviation (0.0273) indicated that model was good and the predicted values will be nearer to actual values of response. The same can be observed from the graph shown in the figure 4.

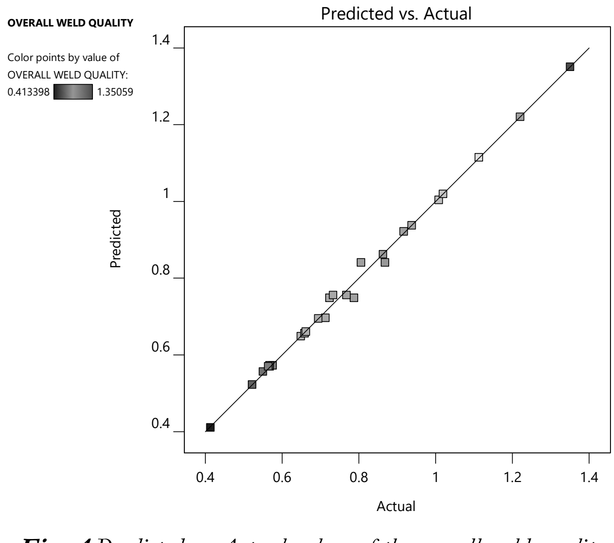

Fig. 4 *Predicted vs Actual values of the overall weld quality*

3.5 Effect of parameters

To study the interaction effect of various combinations of process parameters on the response, the 3D surface plots were used. The 3D response surface plots for significant interactive terms are shown in the figures 5, 6 and 7.

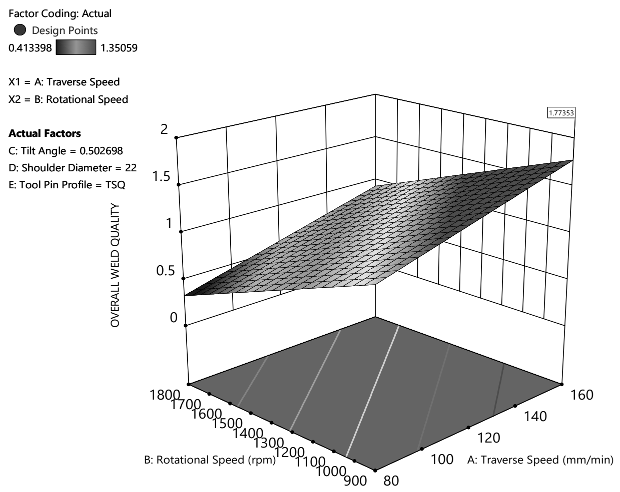

Fig. 5 *Combined effect of traverse speed and rotational speed on overall weld quality*

Figure 5 shows the interaction effect of traverse speed and rotational speed on overall weld quality at constant tilt angle (0.5^0), shoulder diameter (22 mm) and with the taper square tool pin. The two selected factors had contrasting effect. The overall weld quality was increasing gradually with the increase in traverse speed and was decreasing gradually with the increase in rotational speed. Hence the maximum overall weld quality attained at lower limit of rotational speed (900 rpm) and higher limit of traverse speed (160 mm/min).

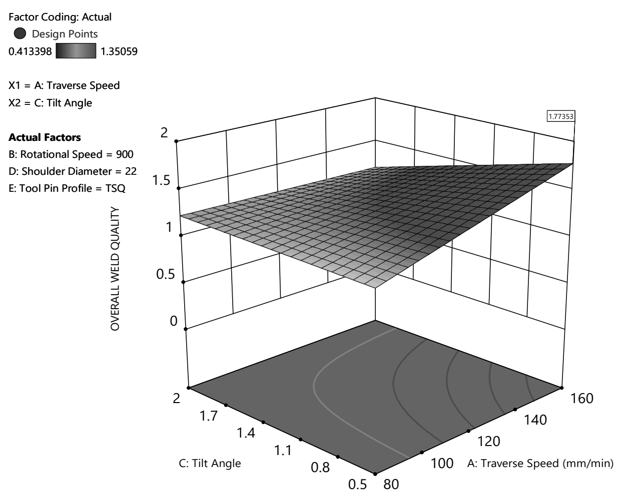

Fig. 6 *Combined effect of traverse speed and tilt angle on overall weld quality*

Figure 6 shows the interaction effect of shoulder diameter and tilt angle on overall weld quality at constant rotational speed (900 rpm), traverse speed (160 mm/min) and with the taper square tool pin. It can be interpreted that at the lower values of traverse speed and higher values of tilt angle the overall weld quality was unaffected by the two parameters. At higher limit of traverse speed the overall weld quality was decreasing with the increase in tilt angle. At lower limit of tilt angle the overall weld quality was increasing gradually with the increase in traverse speed. Hence, the maximum overall weld quality was attained at higher limit of traverse speed (160 mm/min) and lower limit of tilt angle (0.5^0).

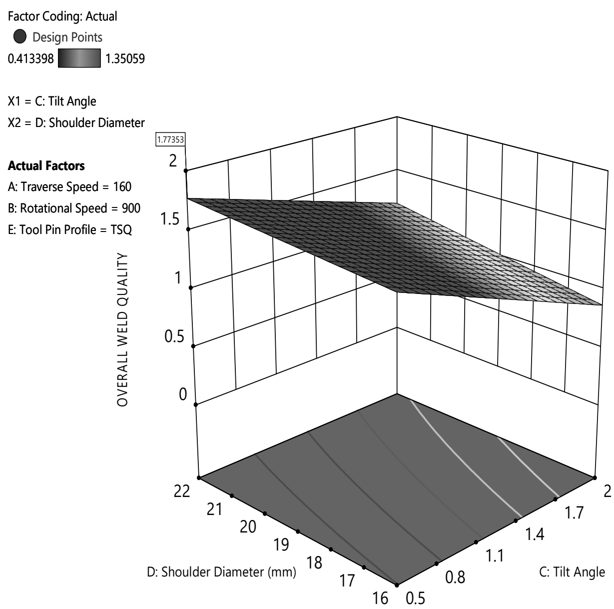

Fig. 7 *Combined effect of shoulder diameter and tilt angle on overall weld quality*

Figure 7 shows the interaction effect of tilt angle and shoulder diameter on overall weld quality at constant traverse speed (160 mm/min), rotational speed (900 rpm) and with taper square tool pin. The overall weld quality was decreasing with the increase in tilt angle and was increasing with the increase in shoulder diameter. Hence, the maximum overall weld quality was attained at lower limit of tilt angle (0.5^0) and higher limit of shoulder diameter (22 mm).

3.6 Confirmation

Numerical optimization was carried by the response surface methodology to maximize the overall weld quality. The optimum process parameters for the maximum overall weld quality of 1.7731 at the desirability value of 0.981 are shown in the table 11.

Tab. 11 *Confirmation test results*

Optimum Process Parameters					Predicted Overall Weld Quality at optimum parameters	Experimental Overall Weld Quality at optimum parameters	% Error	Maximum Overall Weld Quality in Experimental design	% Improvement
TS [mm/min]	RS [rpm]	TA [degrees]	SD [mm]	TPP	1.7735	1.6519	6.85	1.3505	18.25
160	900	0.5	22	TSQ					

Further, confirmation through experiment was carried to validate the developed model. When the welding was carried at optimum process parameters, there was an improvement of 18.25% in overall weld quality over the best performed joint in the experimental design. The values of predicted overall weld quality values and experimental overall weld quality values are in good agreement that affirms the validity of developed GRA-PCA-RSM model.

4 Conclusions

The friction stir welding was successfully carried to join the AA2050 plates with selected range of process parameter values. The hybrid multi-response optimization approach GRA-PCA-RSM adequately developed the regression model for overall weld quality and found the optimum values of process parameters to maximize the overall weld quality.

- The mathematical models for overall weld quality of FS welded joint of AA2050 with different tool pin profiles were developed. The model can predict the overall weld quality within ±10% of the overall weld quality calculated from the experimental results at 95% confidence level.
- Rotational speed and tool pin profile were the most significant parameters. They explain 67.32% and 12.05% of total variation respectively.
- Increase in traverse speed, decrease in rotational speed, decrease in tilt angle and increase in shoulder diameter resulted in improvement of overall weld quality.
- The optimum process parameters to maximize the overall weld quality were obtained from the hybrid approach and are as follows: traverse speed 160 mm/min, rotational speed 900 rpm, tilt angle 0.5^0, shoulder diameter 22 mm and taper square pin profile.

The optimum parametric values of traverse speed and shoulder diameter were at higher limits selected in this study. The optimum parametric values of rotational speed and tilt angle were at lower limits fixed in this study. Further investigation may be carried on FSW of AA2050 plates with parameter values less than lower limits of rotational speeds and tilt angles. Also investigation may be carried with parameter values more than higher limits of traverse speeds and shoulder diameters.

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