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# Spectral Fatigue Life for Simple Notched Component

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**The paper describes a procedure for calculating the lifetime in the frequency domain, which is supplemented by an experiment. The aim of the experiment was to find the required mechanical properties of the material to describe the behavior of the material during fatigue analysis. Experimental modal analysis was performed to estimate damping, which supported numerical calculation and refined results of numerical simulations. The authors made an estimate of fatigue curve (SN-curves) according to ČSN 42 0363 and ČSN ISO 12 107 standards. Knowledge of material fatigue, statistics, modal analysis and signal processing were used in the experimental and numerical part.**

**Keywords:** S-N curve, spectral fatigue, life prediction, Dirlik

## 1 Introduction

The most common cause of machine parts failure is fatigue of the material. This phenomenon occurs as a result of cyclic stressing of the machine part caused by frequent changes in load. In technical practice, we encounter a large number of highly cyclically stressed mechanical components. As the field of uniaxial high-cycle fatigue is relatively well known today, current authors focus more on more complex loading systems or. principles of fatigue life estimation. There are various studies that deal with high-cyclic fatigue from different perspectives, such as fatigue life under multiaxial loading [3],[6],[15],[18],[19] or under different conditions, e.g. elevated temperature [1],[10],[11]. All these tasks can be solved numerically using two approaches - in the time domain and in the frequency domain. The time domain provides a more accurate estimate of fatigue life based on state-of-the-art knowledge, where we obtain a relatively good estimate of life by various linear as well as non-linear criteria. On the other hand, these time approaches are computationally time consuming. If we want to avoid lengthy calculations and save computing capacity, we are currently inclined to use the frequency domain. Many authors [2],[9],[13],[14],[16] have dealt with the principles of frequency life calculation, but it should be noted that this approach loses the accuracy of the calculations. This is due to the fact that the frequency histogram obtained by the Rainflow matrix in estimating the lifetime in the time domain is approximated in the frequency domain by a PDF function, which is obtained based on the parameters of the power spectral density (PSD) function.

## 2 Theory backgroung

Wöhler's "material" curves (Wöhler, 1860), in English often referred to as the S-N curve, shows fatigue life versus the stress amplitude of a smooth material specimen. It can be experimentally determined for different cycle asymmetries which are expressed by the cycle asymmetry factor

$$R = \frac{\sigma_{\min}}{\sigma_{\max}}. \tag{1}$$

The curves of most materials can be very well approximated by Basquin's law

$$\sigma_a = \sigma'_f \left(2N_f\right)^b, \tag{2}$$

Where:

$\sigma'_f$ … fatigue strength coefficient,

$b$ … exponent of fatigue strength,

$2N_f$ … represents the number of half-cycles to failure [20][21].

The most commonly used probability model for calculating fatigue life in the frequency domain is Dirlik model, which discribes a material behaviour at narrow-banding loadning as well as behaviour at wide-banding loading. It is given by formula

$$p(\sigma_a)_{Dirlik} = \frac{\frac{D_1}{Q}\cdot \mathrm{e}^{\frac{-Z_i}{Q}} + \frac{D_2}{R^2}\cdot \mathrm{e}^{\frac{-Z_i^2}{2R^2}} + D_3 \cdot Z_i \cdot \mathrm{e}^{\frac{-Z_i^2}{2}}}{2\cdot\sqrt{m_0}}. \tag{3}$$

Where:

$m_0, m_1, m_2, m_4$ … spectral moments,

$D_1, D_2, D_3, R, Q$ … coeficients,

$Z_i$ … modified stress amplitude [7][12].

Miner's rule of total damage was used to calculate the damage given by the sum of partial damages in form

$$D=\sum_{i=1}^{k}d_i=\sum_{i=1}^{k}\frac{n_i}{N_i}=1. \tag{4}$$

Where:

$D$ … damage,

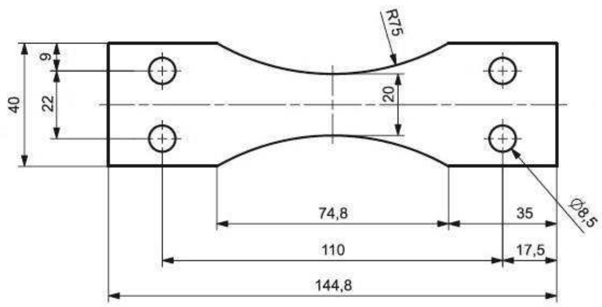
$d_i$ … partial damage,

$N_i$ ⋯ average number of cycles to failure at the i-th stress level,

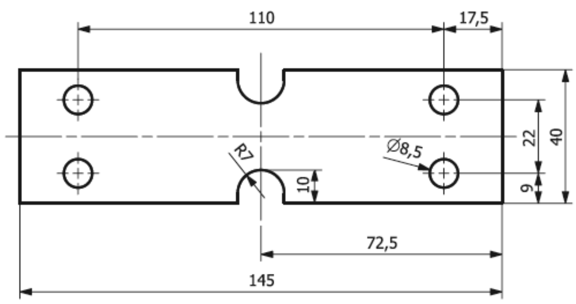
$n_i$ … is the number of cycles accumulated at stress level [8].

### 2.1 Estimation of material SN curves

The SCHENCK PWS 0007 test apparatus was used to create material S-N curves. Fatigue tests were performed as cyclic bending tests on a fully reversed cycle (R = -1) on two types of specimens, specifically smooth (normalized) and notched specimens (Fig. 1). Specimens were made of one piece of aluminum sheet with a constant thickness of 4 mm. The specimen's material was aluminum alloy Al5052 with H32 finish. The tests were carried out at the same temperatures and at 21 ° C.




*a)*




*b)*

***Fig. 1*** *Types of specimen used to creating of material s-n curves. (a) smooth (normalized) specimen for bending fatigue test, (b) specimen with u-shape notch for bending fatigue test.*

The S-N curve for a smooth, unnotched specimen was determined by statistical processing of the data obtained in the bending fatigue test. First specimen was loaded by bending moment which invokes the stress value started at 192.8 MPa after failure. The number of cycles to failure was read and the value was filled in a table for statistical evaluation of the fatigue curve. The other measurements were continued in this way and the load was reduced by approx. 5 MPa per one stress level.

A standard ČSN 42 0363 [4] describes the method of loading and evaluation of specimens, which are subjected to variable amplitude. This standard requires 8 specimens to describe a slop of fatigue curve. In ours measurments were used 20 specimens but some specimens had to be eliminated from testing group due to wrong measurement. To statistical proccesing was used the standard ČSN ISO 12 107 [5], which offers three types of fatigue curve aproximation. The linear aproximation (2) describes by Basquin formula was choosen.

$$\log N = a + b\sigma\,, \tag{5}$$

Where:

$$a=\frac{1}{n}\sum_{i=1}^{n}\log N_i - b\frac{1}{n}\sum_{i=1}^{n}\sigma_i\,, \tag{6}$$

$$b=\frac{\frac{1}{n}\sum_{i=1}^{n}\left(\log N_i\right)\sigma_i-\sum_{i=1}^{n}\sigma_i\sum_{i=1}^{n}\log N_i}{n\sum_{i=1}^{n}\sigma_i^2-\left(\sum_{i=1}^{n}\sigma_i\right)^2}\,, \tag{7}$$

The shape and results of the S-N curves for the notched and smooth specimen are shown in Fig. 2 and values necessary for numerical calculation are filled in **Tab.** .

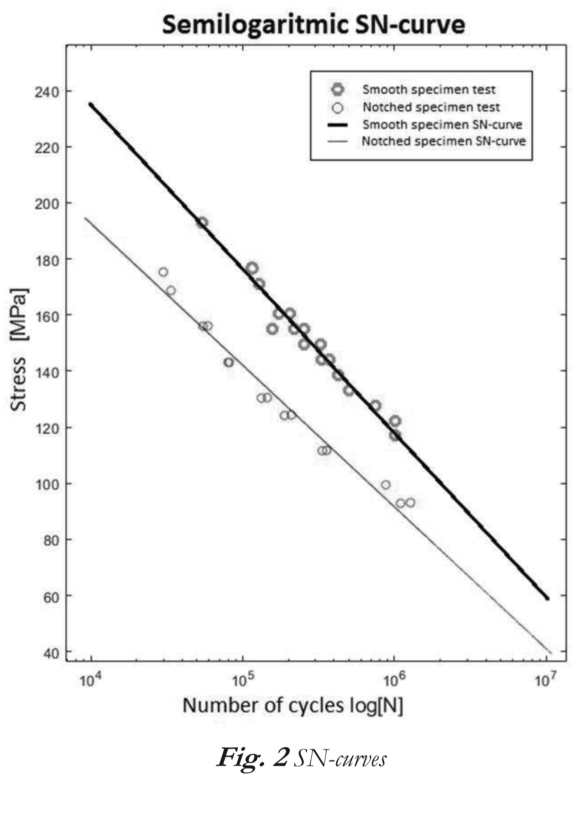



***Fig. 2*** *SN-curves*

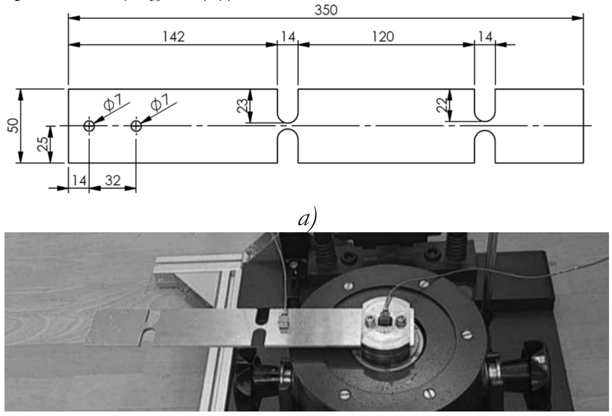
***Tab. 1*** *Values read from fatigue curve for smooth specmen*

| Number of cycles | Smooth specimen (Stress [MPa]) | Notched specimen (Stress [MPa]) |
|---|---|---|
| 10 000 | 234.7 | 198.2 |
| 50 000 | 191.9 | 160.7 |
| 100 000 | 175.4 | 144.6 |
| 250 000 | 153.5 | 123.3 |
| 500 000 | 137.0 | 107.1 |
| 1 000 000 | 117.9 | 91.0 |
| 1 500 000 | 110.8 | 81.6 |

The results clearly show that the notched specimen is weakened due to the notch. It could be seeing in Tab. 1, because a decrease in the fatigue life (number of cyclesto failure) is approximately about 20% for the notched specimen.Therefore, it is necessary to use the fatigue curve of notched specimen if we are working with notched construction or we should be use some correction coeficient.

### 2.2 Fatigue test

Ten test specimens (Fig. 3 (a)) were subjected to a random vibration fatigue test. Before starting the measurement, it was necessary to set the appropriate amplitude on the input excitation signal, which generated white noise in the range 25-800 cycles per second. Two uniaxial acceleration sensors were placed on the specimen (Fig. 3 (b)).




*a)*

*b)*

***Fig. 3*** *a) shape and dimensions of specimen, (b) measuring set prepared to experiment*

All specimens were tested under the same conditions. The time to failure (Tab. 2) ranged from 815 to 1470 s. The standard deviation at this interval was 175.9 s. The average time to fatigue failure of the test specimens at the oscillation is 1095 s (18 min and 25 sec). The location of the failure (Fig. 6) was identical in all cases.

***Tab. 2*** *Time to specimens damage*

| Serial num. | Spec. num. | Damage time | Serial num. | Spec. num. | Damage time |
|---|---|---|---|---|---|
| 01 | 03 | 1040s(17min 20s) | 06 | 09 | 820s(13min 40s) |
| 02 | 05 | 1445s(24min 05s) | 07 | 10 | 1132s (18min 52s) |
| 03 | 06 | 1000s(16min 40s) | 08 | 11 | 1150s (19min 10s) |
| 04 | 07 | 815s (13min 35s) | 09 | 12 | 1470s (24min 30s) |
| 05 | 08 | 1022s(17min 02s) | 10 | 13 | 1058s (17min 38s) |

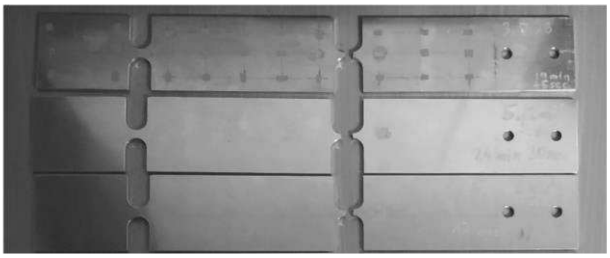


***Fig. 4*** *Fatigue damage of specimens.*

### 2.3 Modal analysis

Modal analysis (Fig. 5) was performed on fatigue test specimens to verify the natural frequencies and to determine damping. The damping ratios obtained experimentally were used in the simulation to refine the response of the numerical model. Experimental modal analysis was performed using PULSE system. The responses were measured by Polytec PDV-100 laser vibrometer and the structure was excited by Bruel & Kjaer 8206 modal hammer. The selected modal parameters corresponding to bending shapes are shown in Tab. 3.

The numerical modal analysis was performed in program CAE Abaqus. The obtained eigenfrequencies are also shown in Tab. 3.

Differencies in values of eigenfrequencies were caused by ideal fixation of the specimen in numerical simulations. It may have a small negative effect for fatigue calculation.

***Tab. 3*** *Eigenfrequencies and damping of the test specimen, corresponding to bending shapes.*

| Eigenmode | Eigenfrequency (FEM) [Hz] | Damped eigenfrequency (EMA) [Hz] | Damping ratio [%] | Mode complexity |
|---|---|---|---|---|
| 1. | 31.08 | 31.80 | 1.22684 | 0.01680 |
| 3. | 217.65 | 191.68 | 0.45291 | 0.00278 |
| 5. | 498.69 | 472.87 | 0.22234 | 0.00074 |
| 8. | 1114.7 | 1081.90 | 0.09551 | 0.00378 |

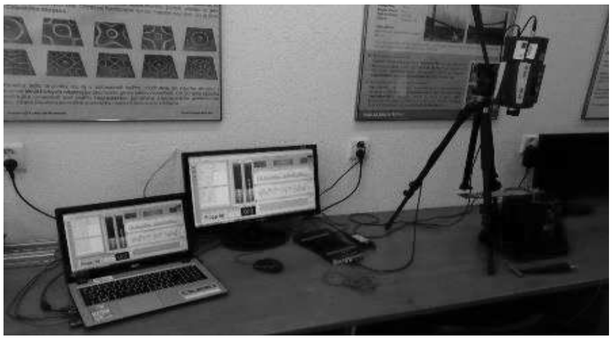
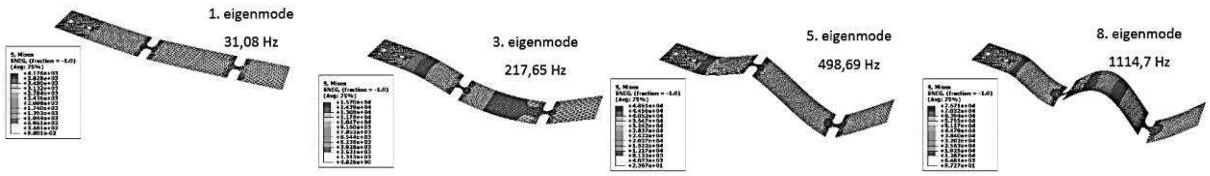


***Fig. 5*** *EMA - measurement process.*

## 3 FEM-simulation

Linear dynamic analysis was performed in CAE Abaqus. The model was excited by acceleration in one direction, which was applied perpendicular to the largest surface and the accelerometer location was at the same location as the acceleromater location in the experiment.

The lifetime calculation in frequency domain was performed in FE-SAFE, where the Dirlik method was chosen, which generally covers calculations of both narrowband and broadband signals. Modal stresses (Fig. 6 (a)) obtained in modal analysis, generalized phase angle shifts, obtained in frequency analysis using ABAQUS software were used as output data for fatigue life calculation. The S-N curve of the notched specimen was used in the calculation to give a more accurate estimate. The resulting field of damage in logarithmic values can be seen at (Fig. 6 (b)), showing that the most damaged place is consistent with the experiment.

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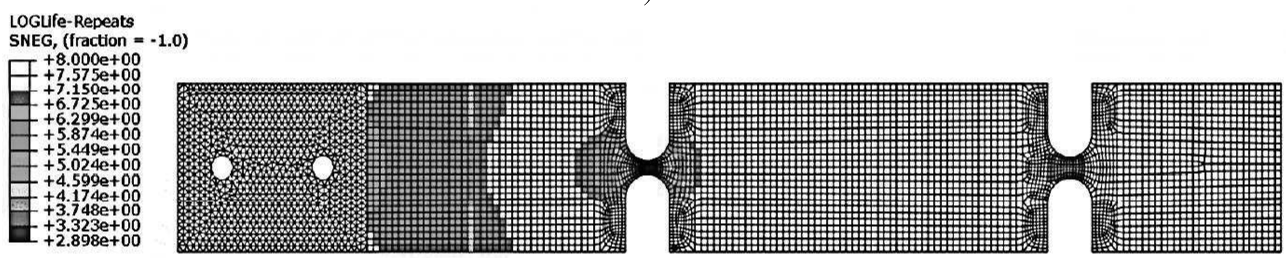


*(b)*

***Fig. 6*** *(a) First four bending eigenmodes. (b) Damage field of investigated specimen.*

## 4 Conclusion

2x20 specimens were subjected to bending tests to obtain fatigue characteristics. The obtained characteristics were used in the numerical calculation. 10 specimens were subjected to the fatigue life test of a notched beam under the dynamic loading. The average time to failure of the beam was determined by the experimental analysis at 1095 seconds. The numerical analysis of the fatigue life using the Dirlik method was 791 seconds and for the Tovo-Benasciutti method 796 seconds. In both cases, the S-N curve of the notched specimen was considered, due to more accuracy. The differences in the results can be attributed to the ideal rigid of the specimens in the FEM program, the approximation of the PSD signal, the damping of the individual eigenmodes, and the accuracy of the S-N curves.

## Acknowledgement

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